Asiacrypt 2023

ANTRAG: Annular NTRU Trapdoor Generation

Making Mitaka as secure as Falcon

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NIST
National Institute of Standards and Technology
Center of Excellence

Post-quantum Hash-and-Sign over lattices

Falcon (*NIST 2017*) 

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- Short signature
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ANTRAG: Make Mitaka as secure as Falcon

Hash-and-Sign over lattices

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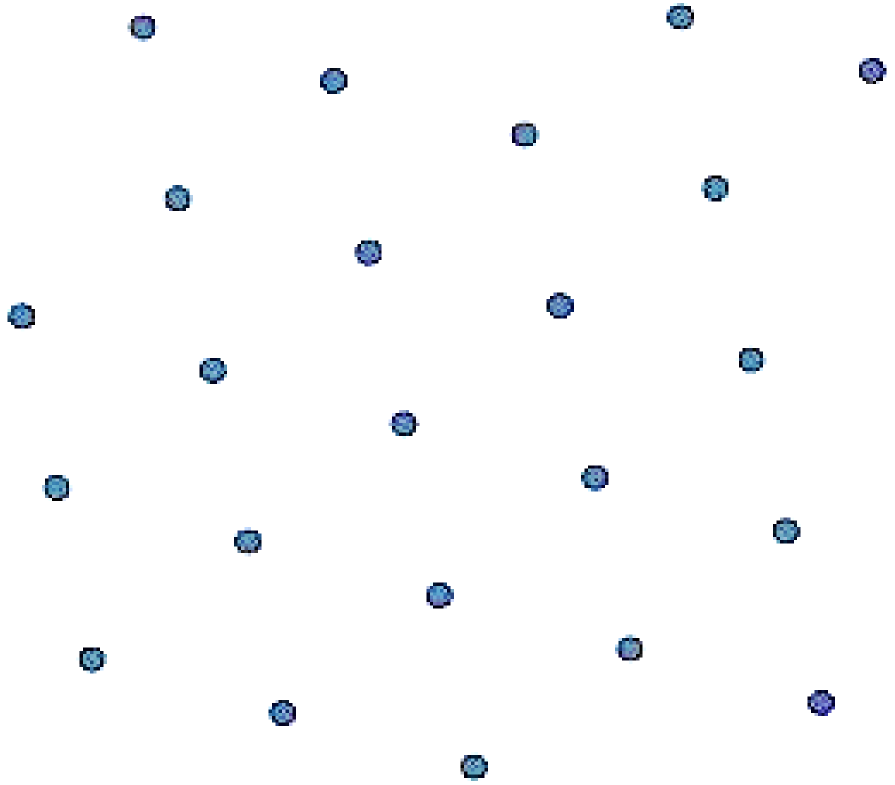
Sign($\mathbf{m}$, $\mathbf{sk}_{\Lambda}$, γ):



$\Lambda \subset \mathbb{R}^d$

Hash-and-Sign over lattices

Sign$(\mathbf{m}, \mathbf{sk}_\Lambda, \gamma)$:

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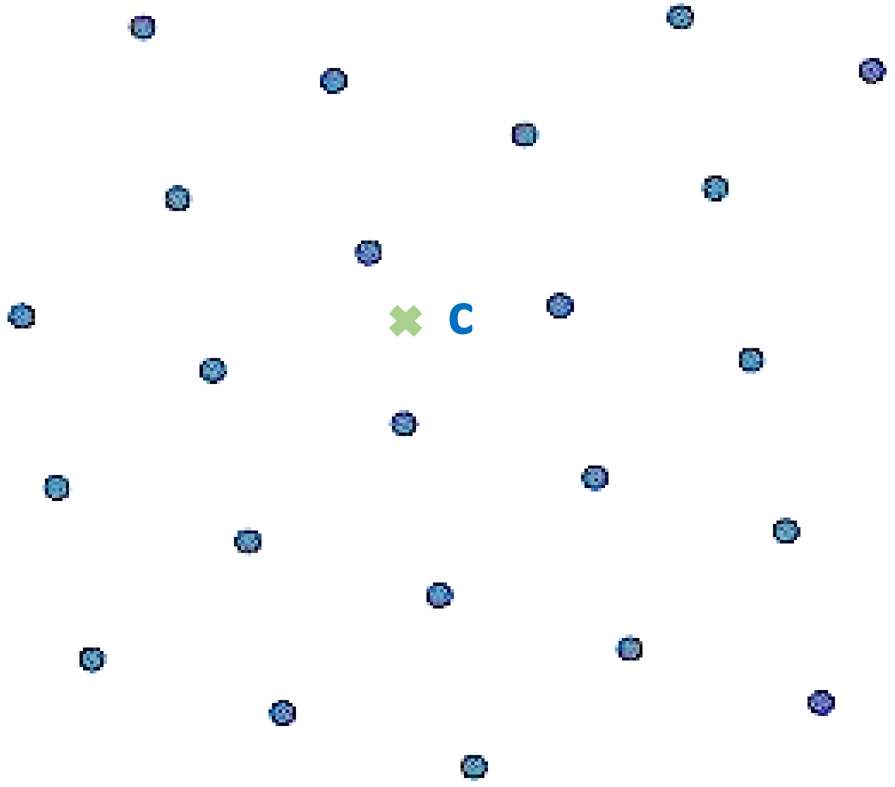

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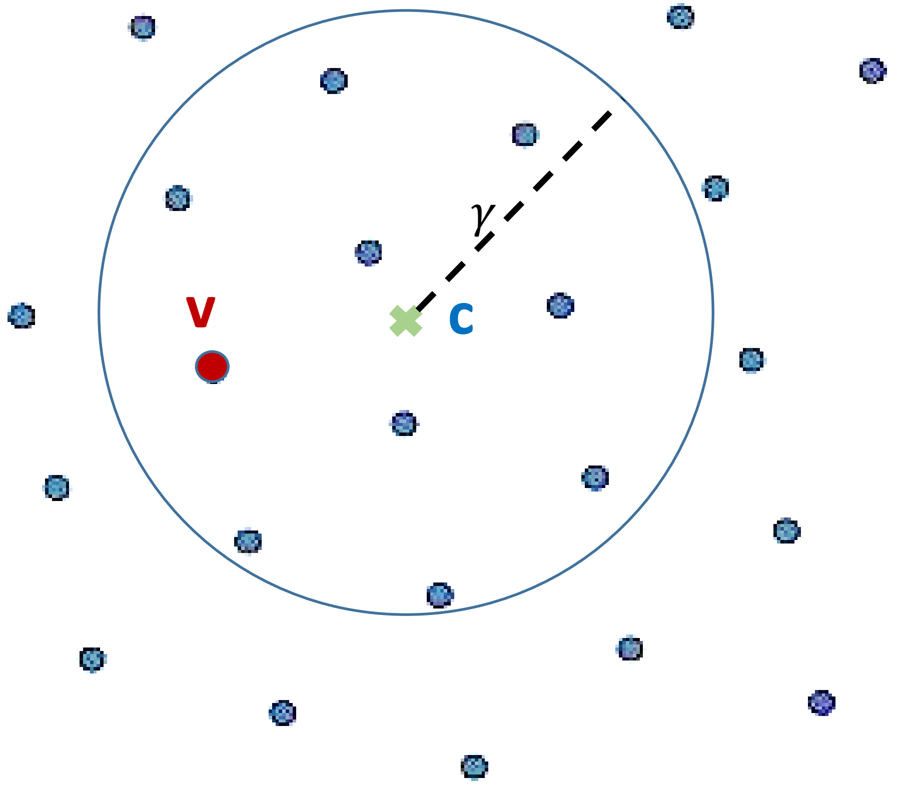

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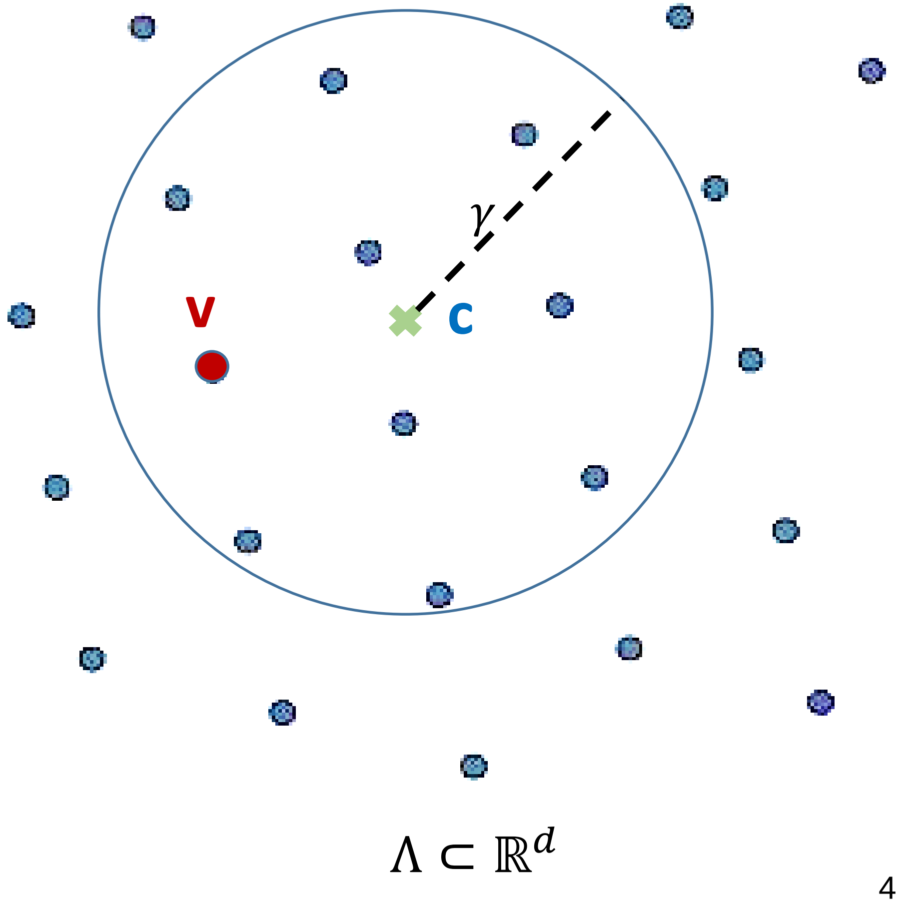

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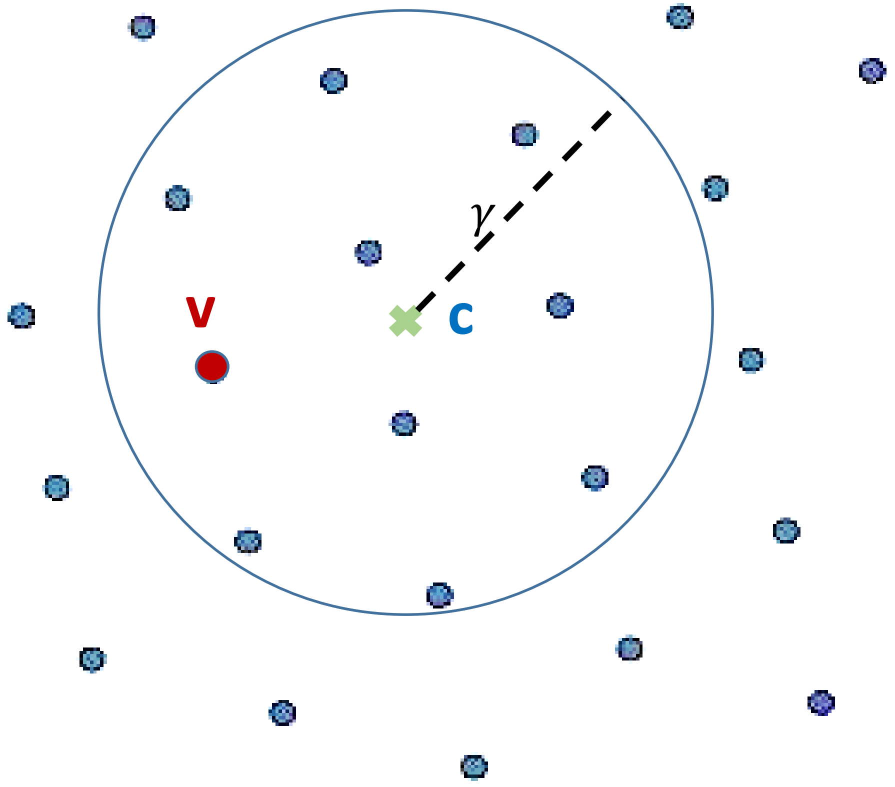

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Hash-and-Sign over lattices

Sign$(\mathbf{m}, \mathbf{sk}_\Lambda, \gamma)$:

- $\mathbf{c} := H(\mathbf{m})$
- $\mathbf{v} \leftarrow \text{DiscreteGaussianSampler}(\mathbf{sk}_\Lambda, \mathbf{c})$
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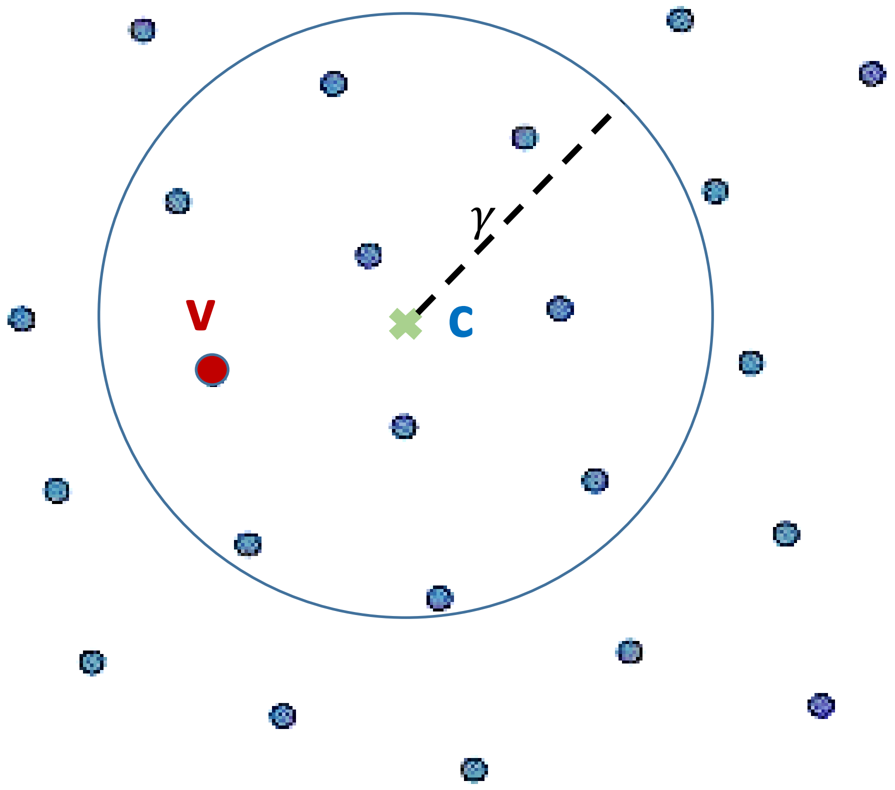
CVP_γ



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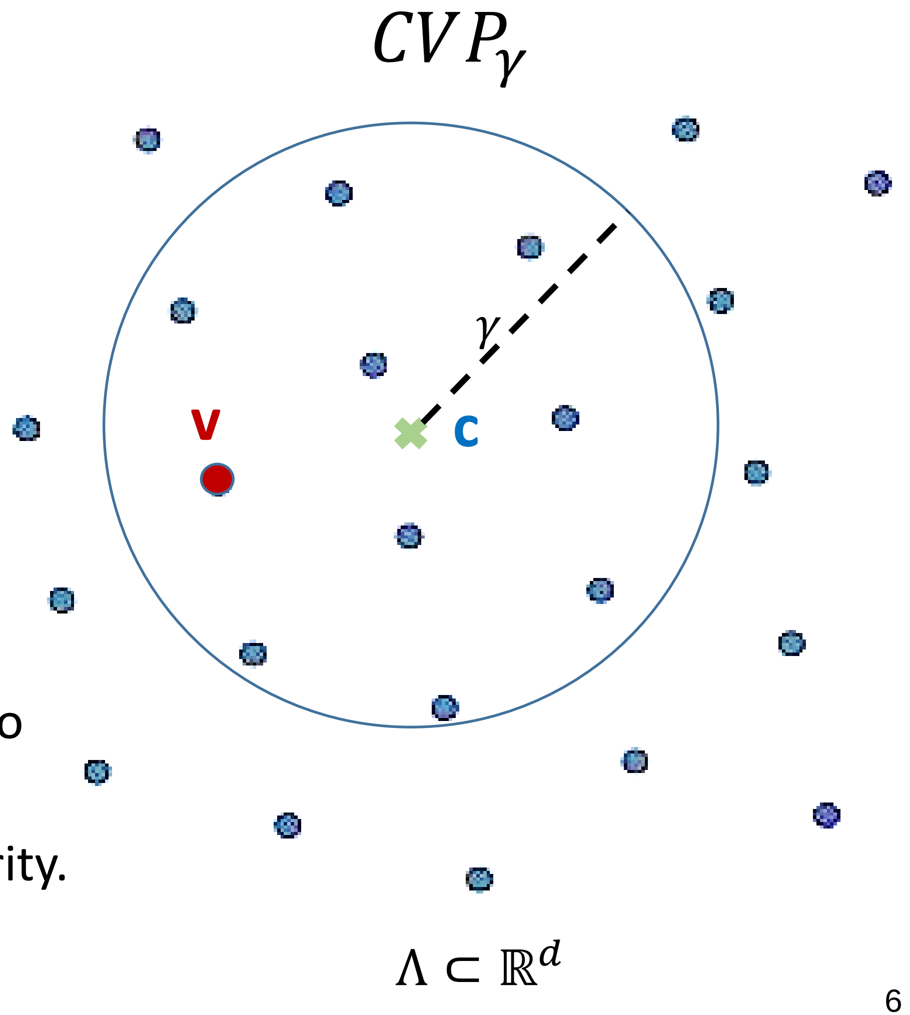

Remarks:

- **Security** : related to Close Vector Problem (CVP) hard to solve without $\mathbf{sk}$.
- Smaller $\mathrm{DiscreteGaussianSampler}(\mathbf{sk},\cdot)$: better security.

→ need $\mathbf{sk}$ of « good quality ».

NTRU lattices

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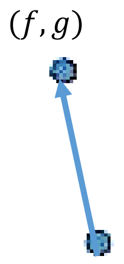

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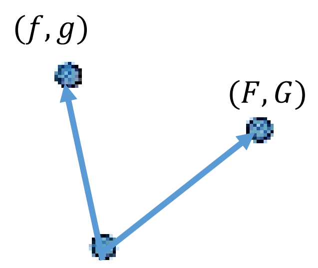

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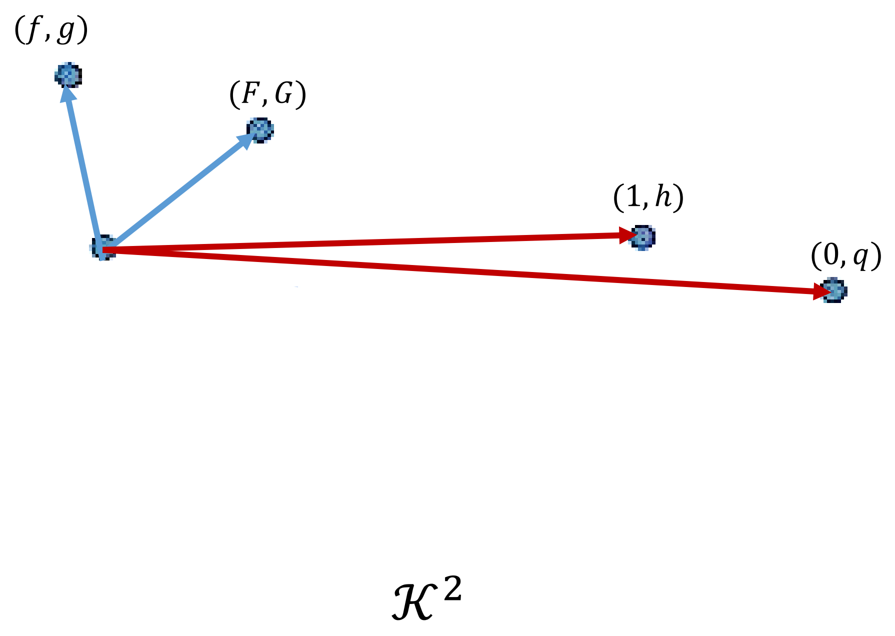

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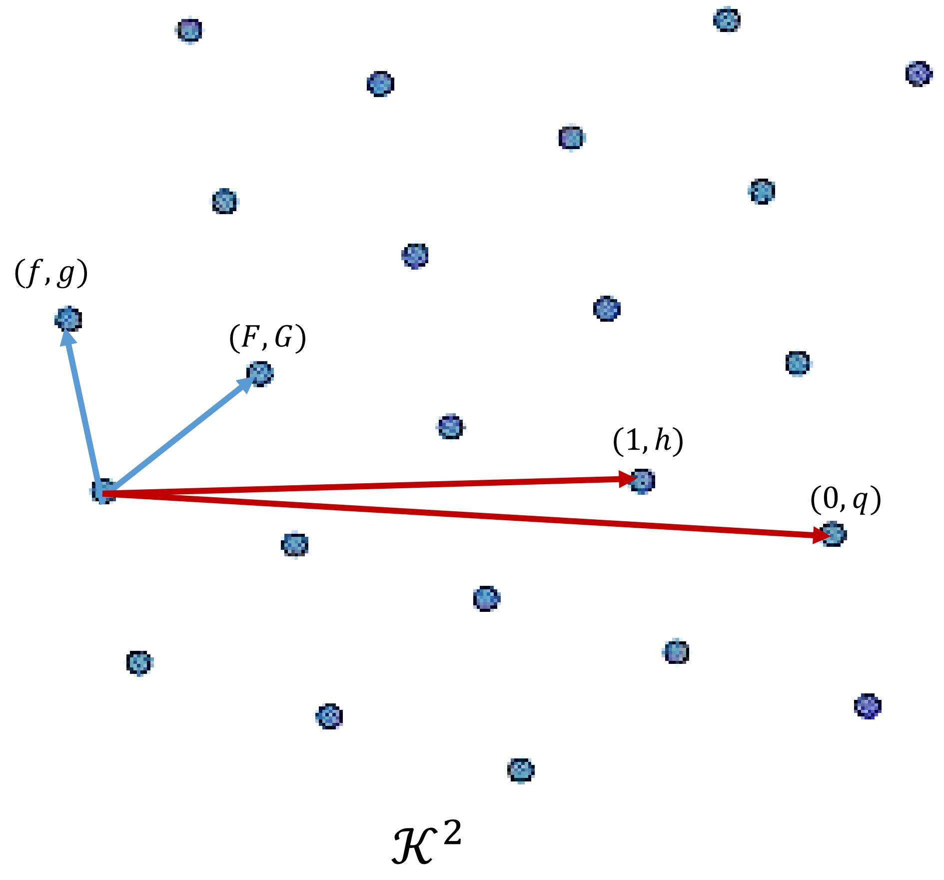

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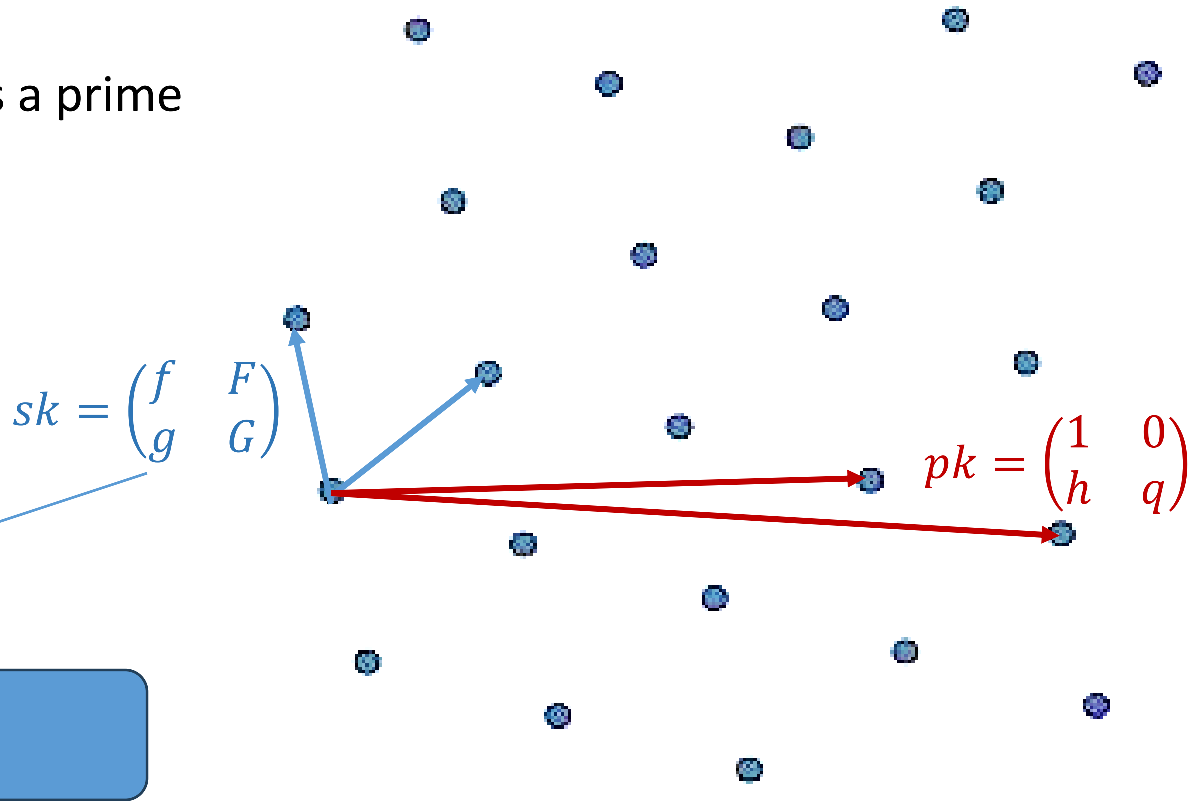
- The secret key sk is the trapdoor.

NTRU ***Trapdoor*** generation

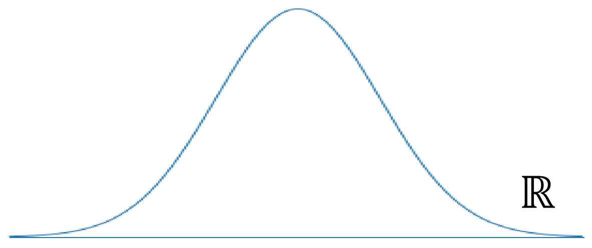


$\Lambda_{NTRU} \subset \mathbb{Z}^{2n}$

Gaussian Distributions

Gaussian Distributions

- Gaussian Distribution $\mathcal{N}_{\mathbb{R},c,\sigma}$



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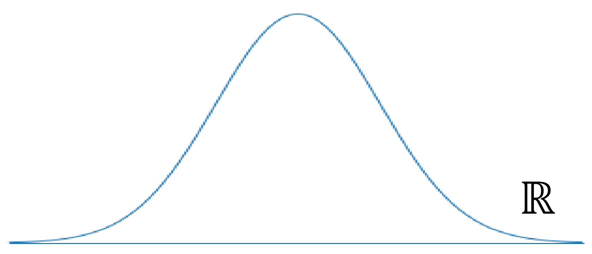
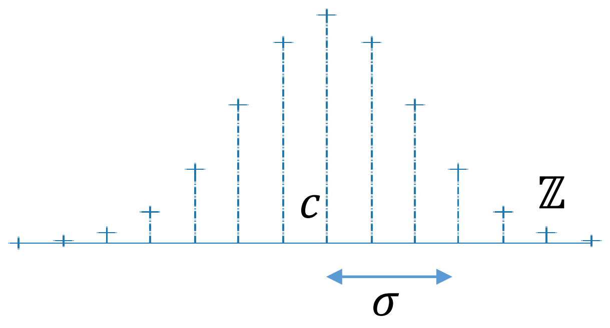


- Discrete Gaussian Distribution on $\mathbb{Z}$: $D_{\mathbb{Z},c,\sigma}$



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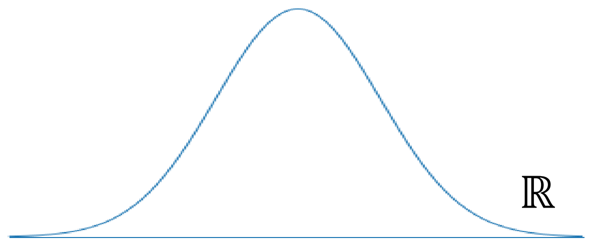

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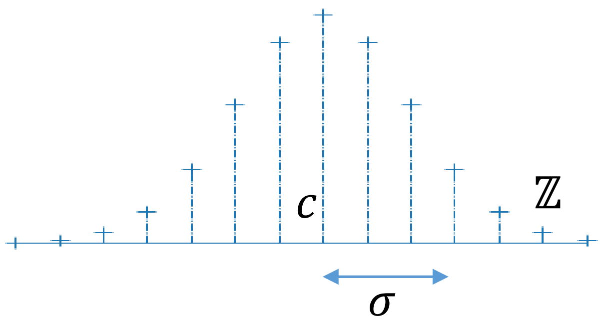
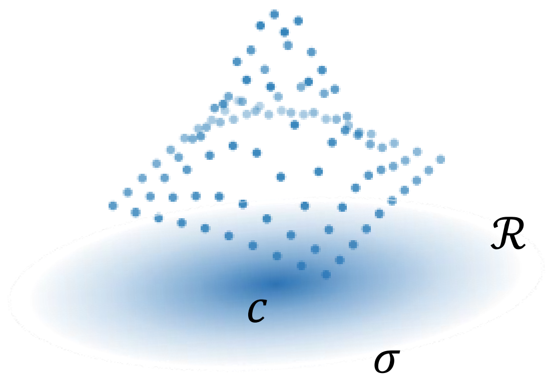


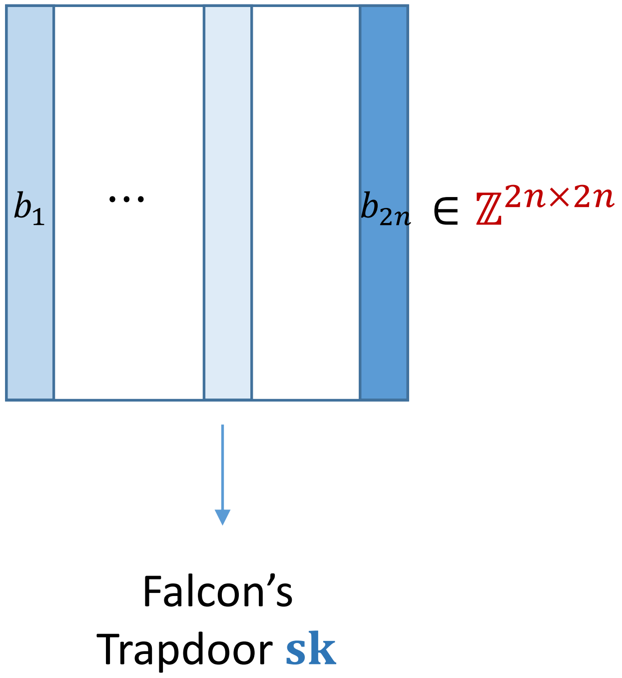
- Discrete Gaussian Distribution on Ring $\mathcal{R}$: $D_{\mathcal{R},c,\sigma}$



DiscreteGaussianSampler$(\mathbf{sk}_\Lambda, \mathbf{c}) \to \mathbf{v}$

DiscreteGaussianSampler($\mathbf{sk}_\Lambda, \mathbf{c}$) $\rightarrow \mathbf{v}$

KGPV sampler
[Kle00,GPV08]



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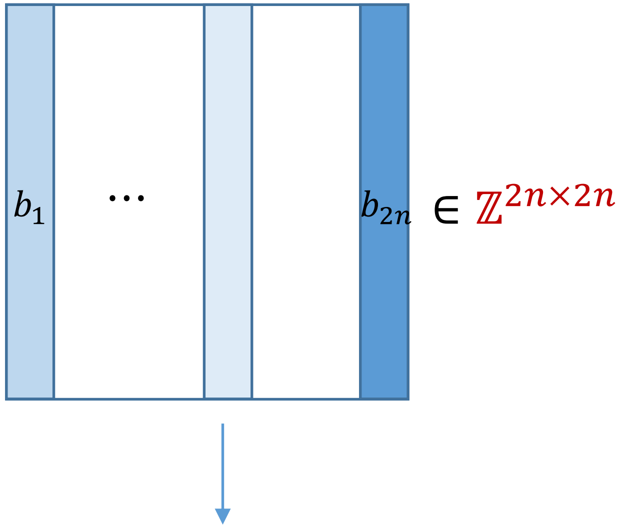
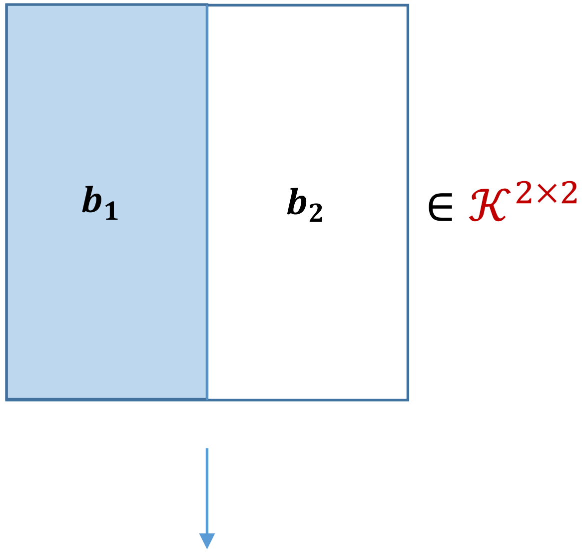


Falcon’s
Trapdoor **sk**

Hybrid sampler
[Pre15]



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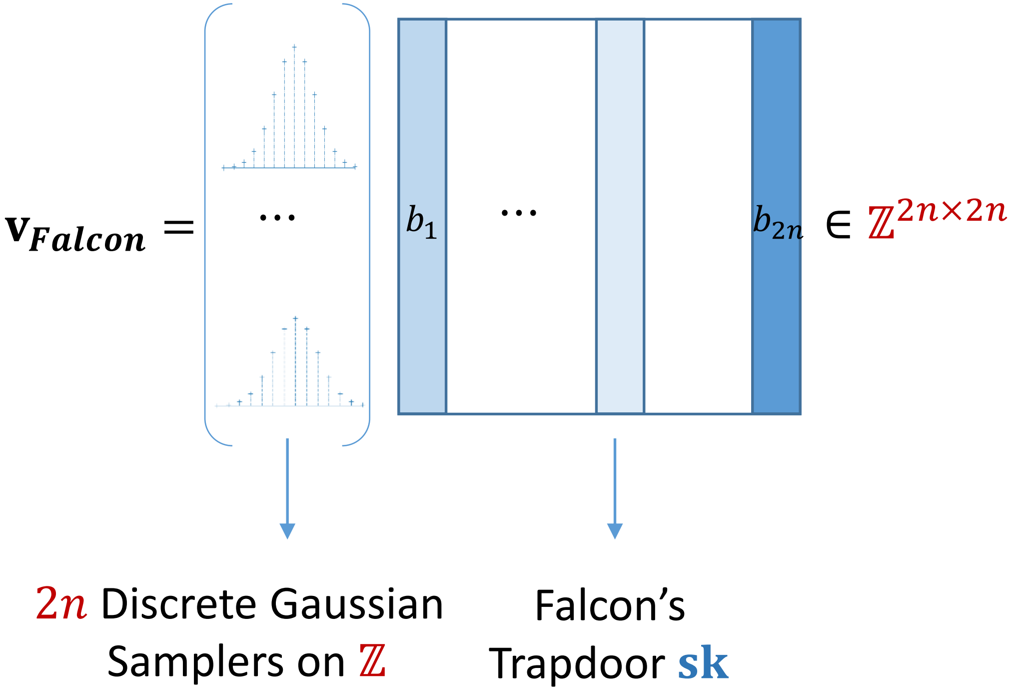

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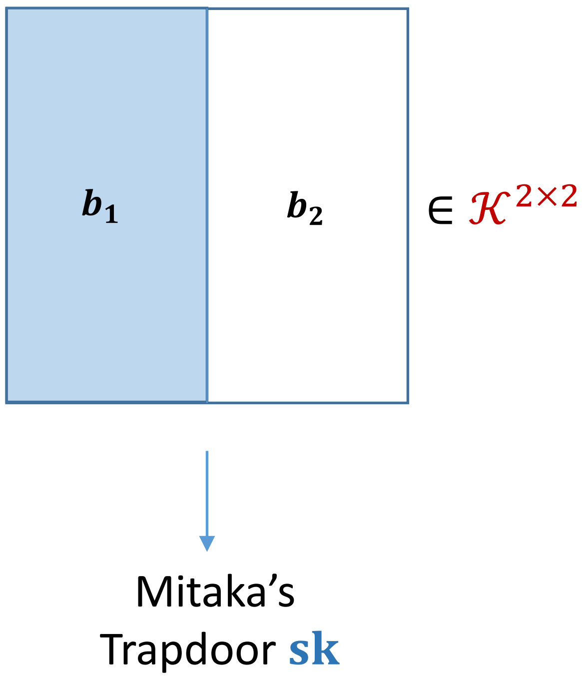

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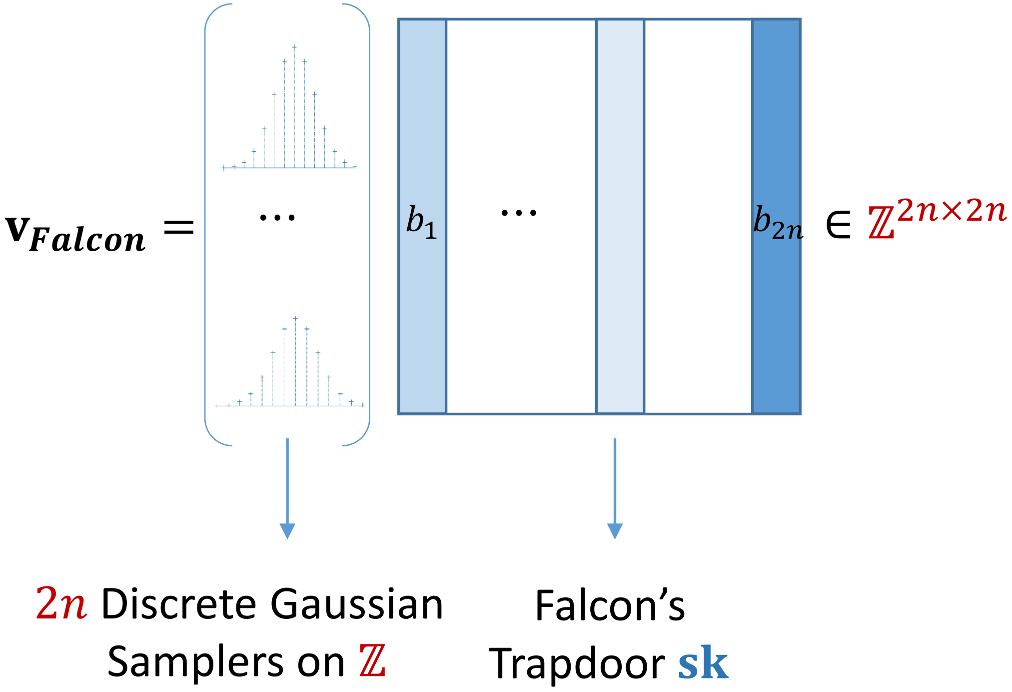

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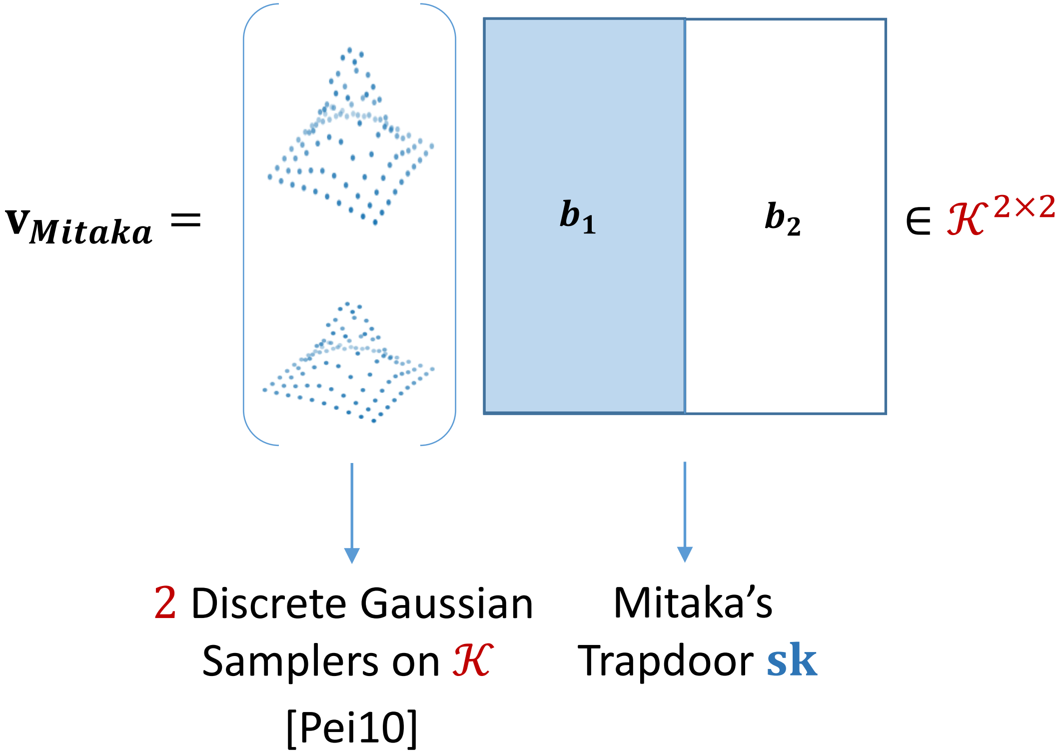

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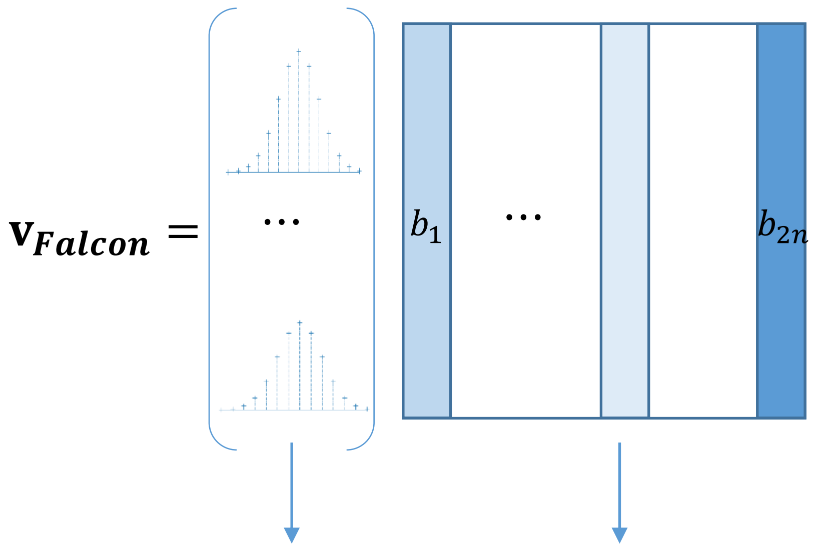
KGPV sampler

Quadratic

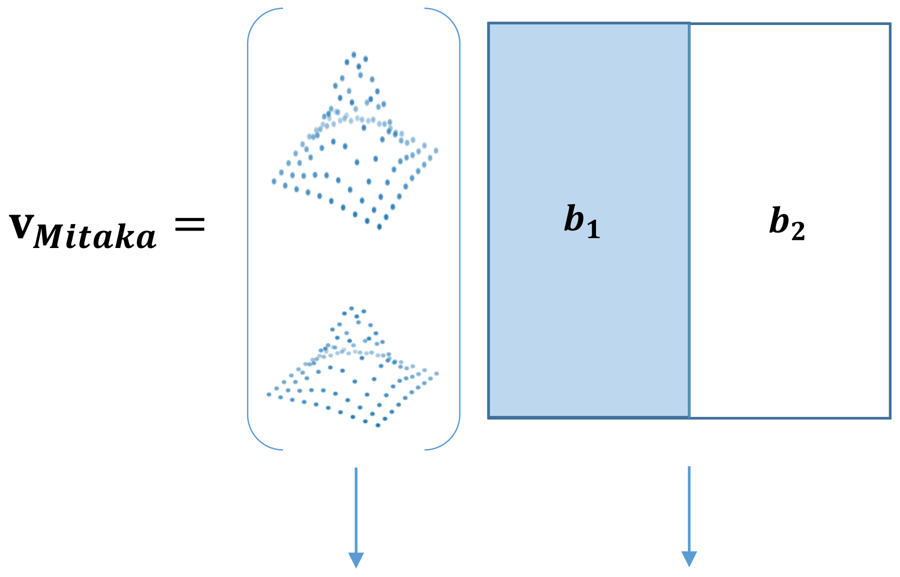


$2n$ Discrete Gaussian Samplers on $\mathbb{Z}$

Falcon's Trapdoor $\mathbf{sk}$

Hybrid sampler

Quasi-linear



2 Discrete Gaussian Samplers on $\mathcal{K}$ [Pei10]

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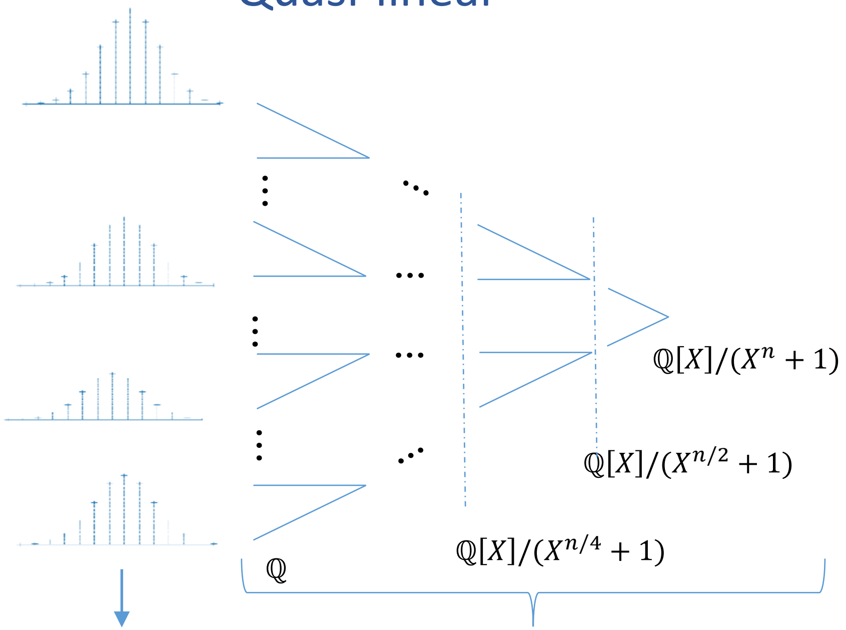
FFO sampler [DP16]

Quasi-linear

$\mathbf{v}_{Falcon} =$



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Falcon's tree: complex Trapdoor **sk**

Hybrid sampler

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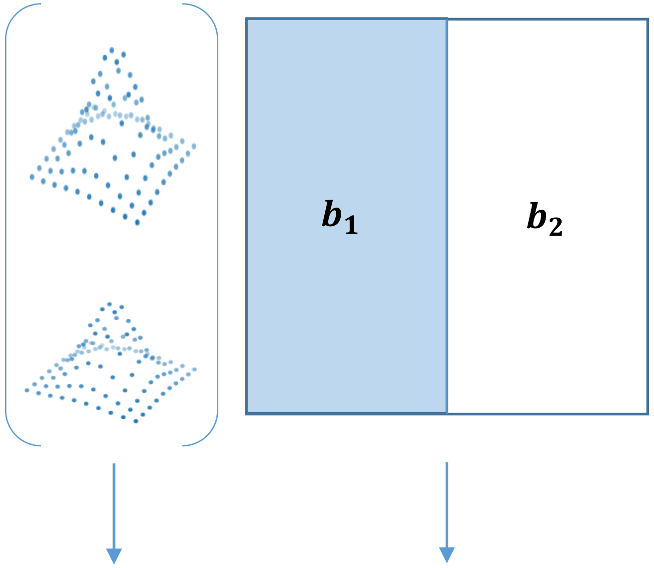

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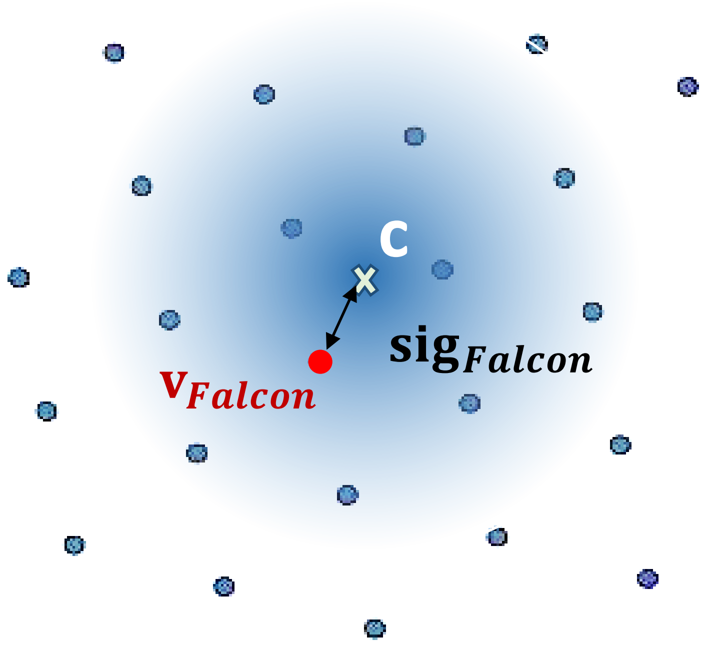
Mitaka's Trapdoor **sk**

Sampler/Signature's size

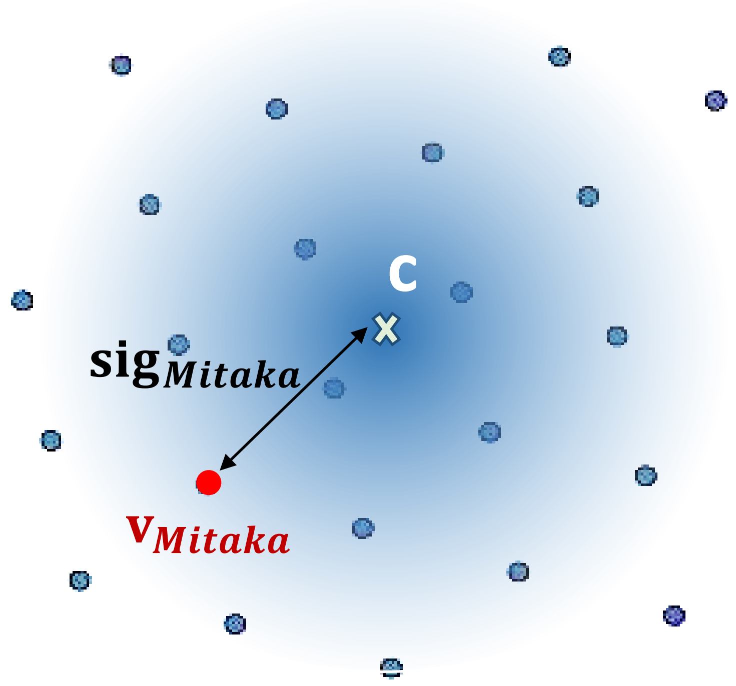
Falcon



$$\|\mathbf{sig}_F\| \propto \|\mathbf{sk}\|_{Klein} \approx 1.17\sqrt{q}$$

$<$

Mitaka



$$2.04\sqrt{q} \approx \|\mathbf{sk}\|_{hybrid} \propto \|\mathbf{sig}_M\|$$

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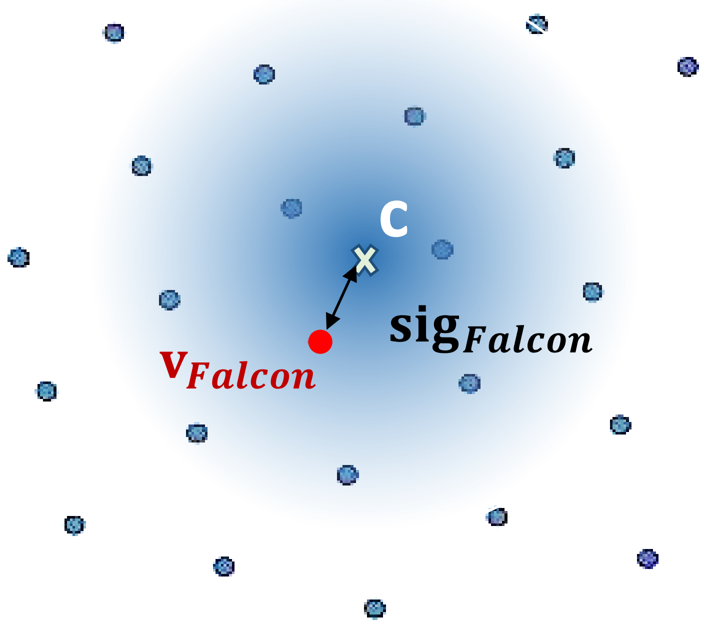
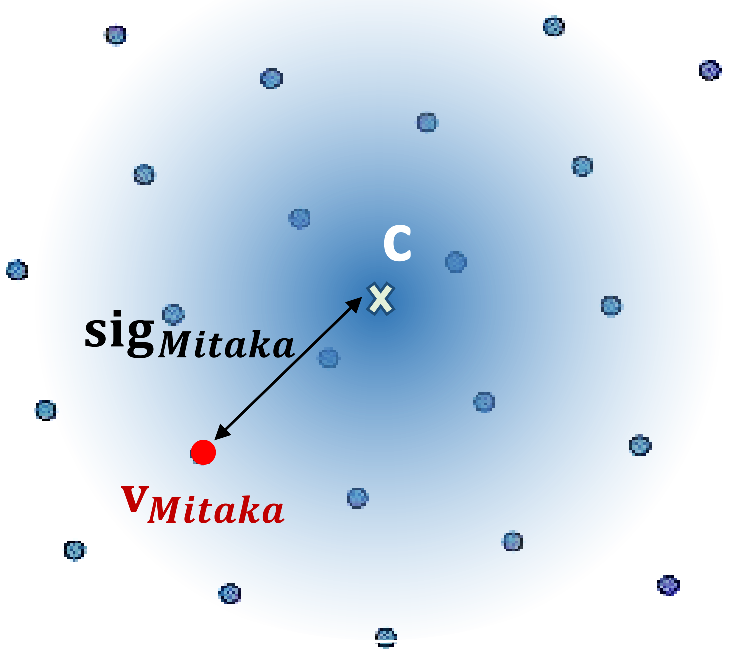


Mitaka



Falcon < Mitaka

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Quality α

Quality α and Trapdoor Generation

The security of the scheme depends on the quality α of the **trapdoor**

$$\alpha = \frac{\|\mathbf{sk}\|}{\sqrt{q}} = \frac{1}{\sqrt{q}} \left\| \begin{pmatrix} f & F \\ g & G \end{pmatrix} \right\|$$

with $\|\cdot\|$ defined by the **sampler** .

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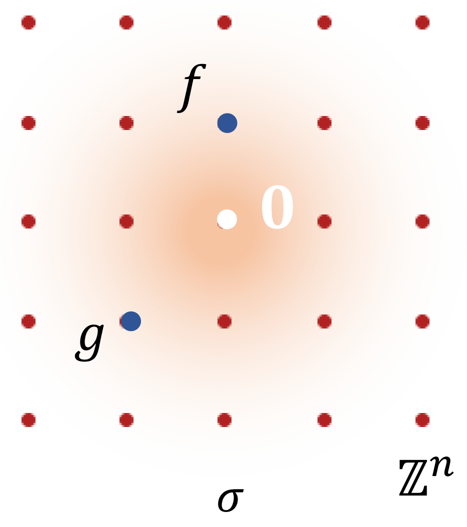

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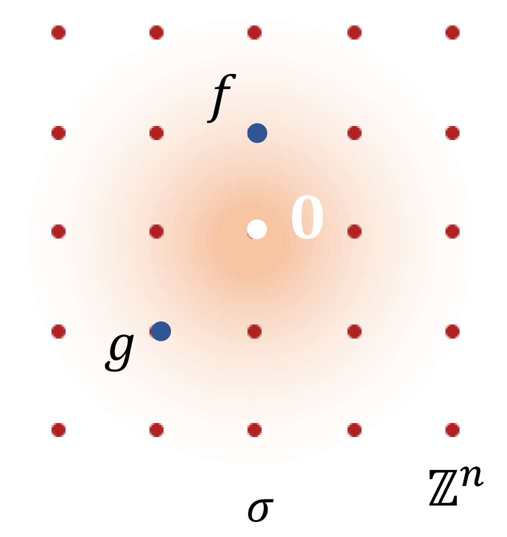

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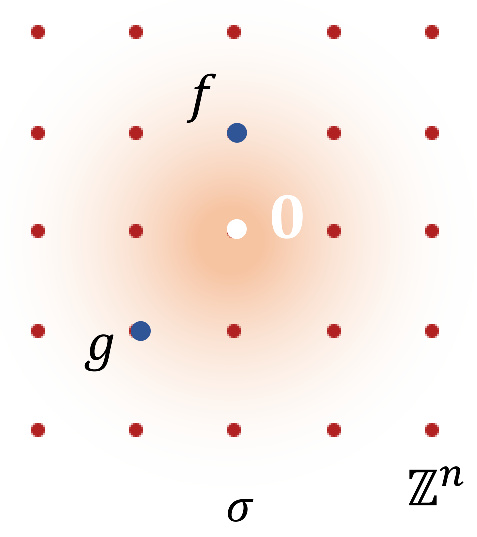
› Our method:



ANTRAG: Annular Trapdoor Generation for Mitaka
$\alpha_{Mitaka} = 1.15$

ANTRAG: Annular NTRU Trapdoor Generation

$$\mathbb{Z}^n \approx \mathcal{K} \ni \sum_n f_i x^i = f \xrightarrow{\text{DFT}} \big(f(\zeta_1), \cdots, f(\zeta_n)\big) \in \mathbb{C}^n$$

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- For fixed $\alpha_{Mitaka} = \alpha$, we want to find f, g such that for $\forall i \leq n$

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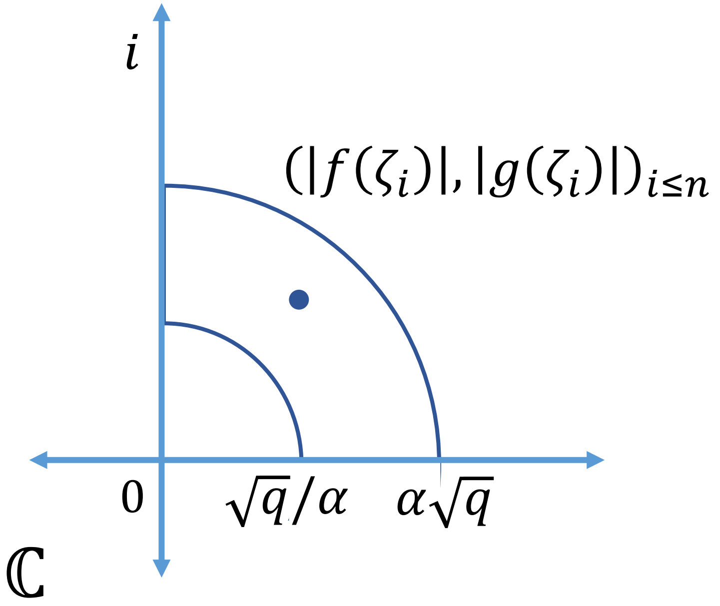
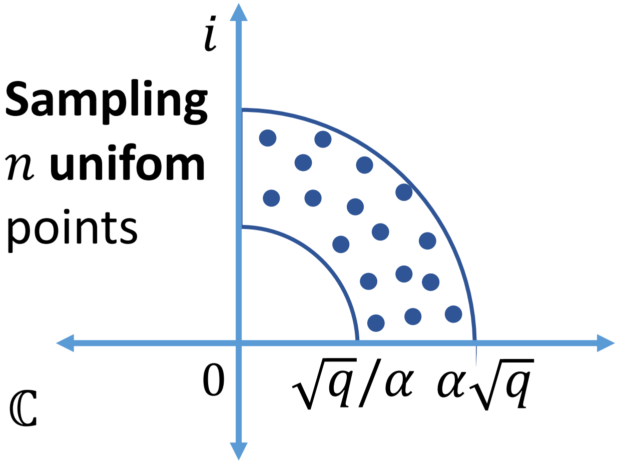


ANTRAG: Annular NTRU Trapdoor Generation (1)

DFT representation



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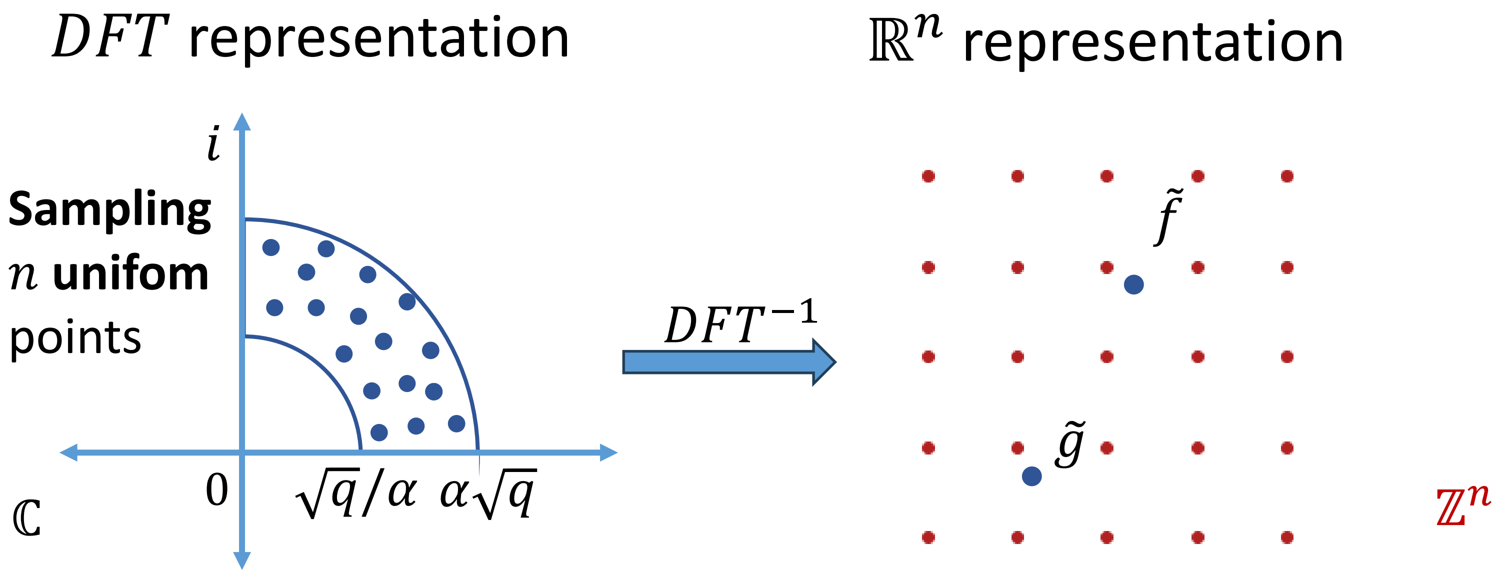
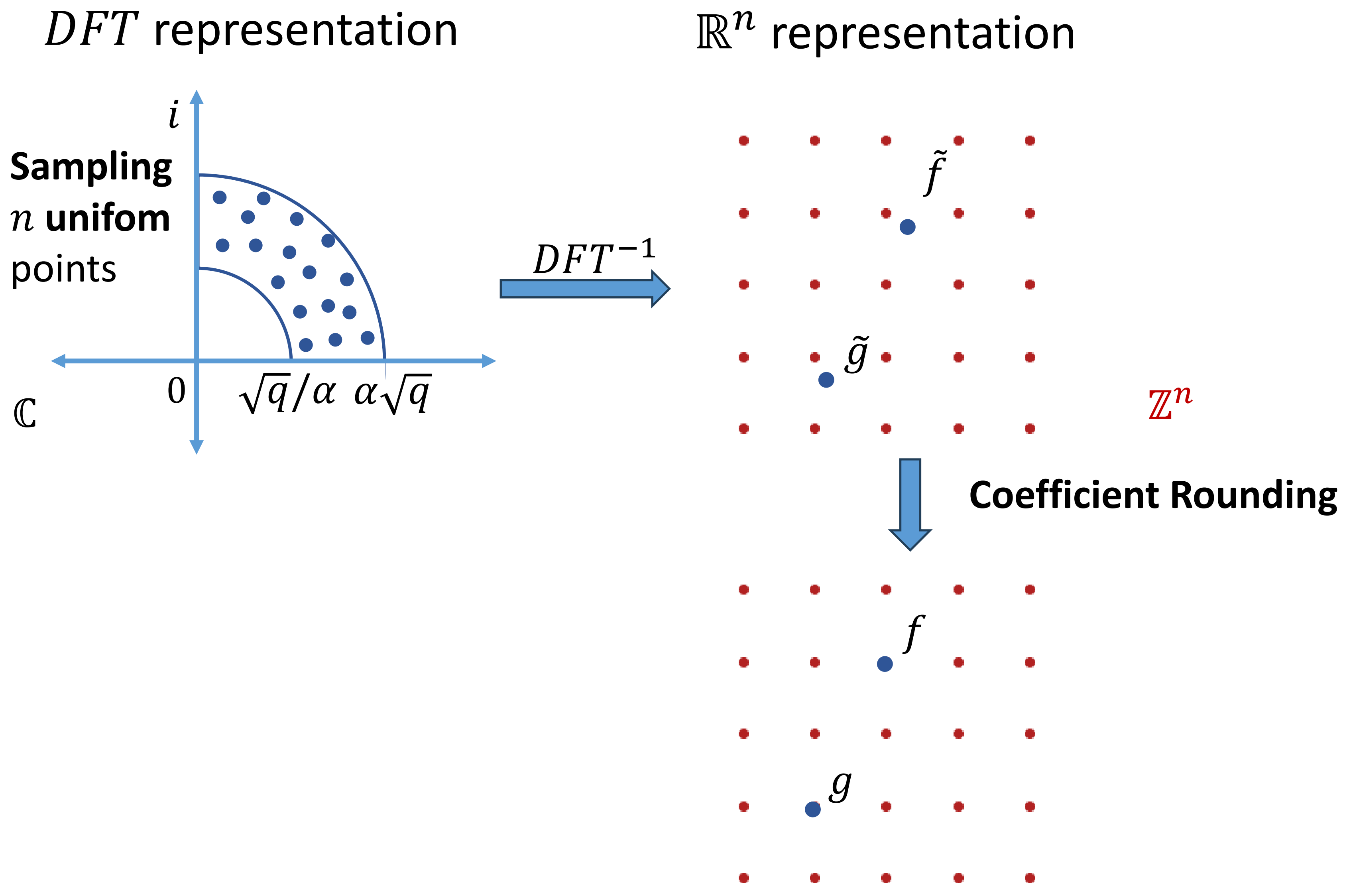


ANTRAG: Annular NTRU Trapdoor Generation (1)



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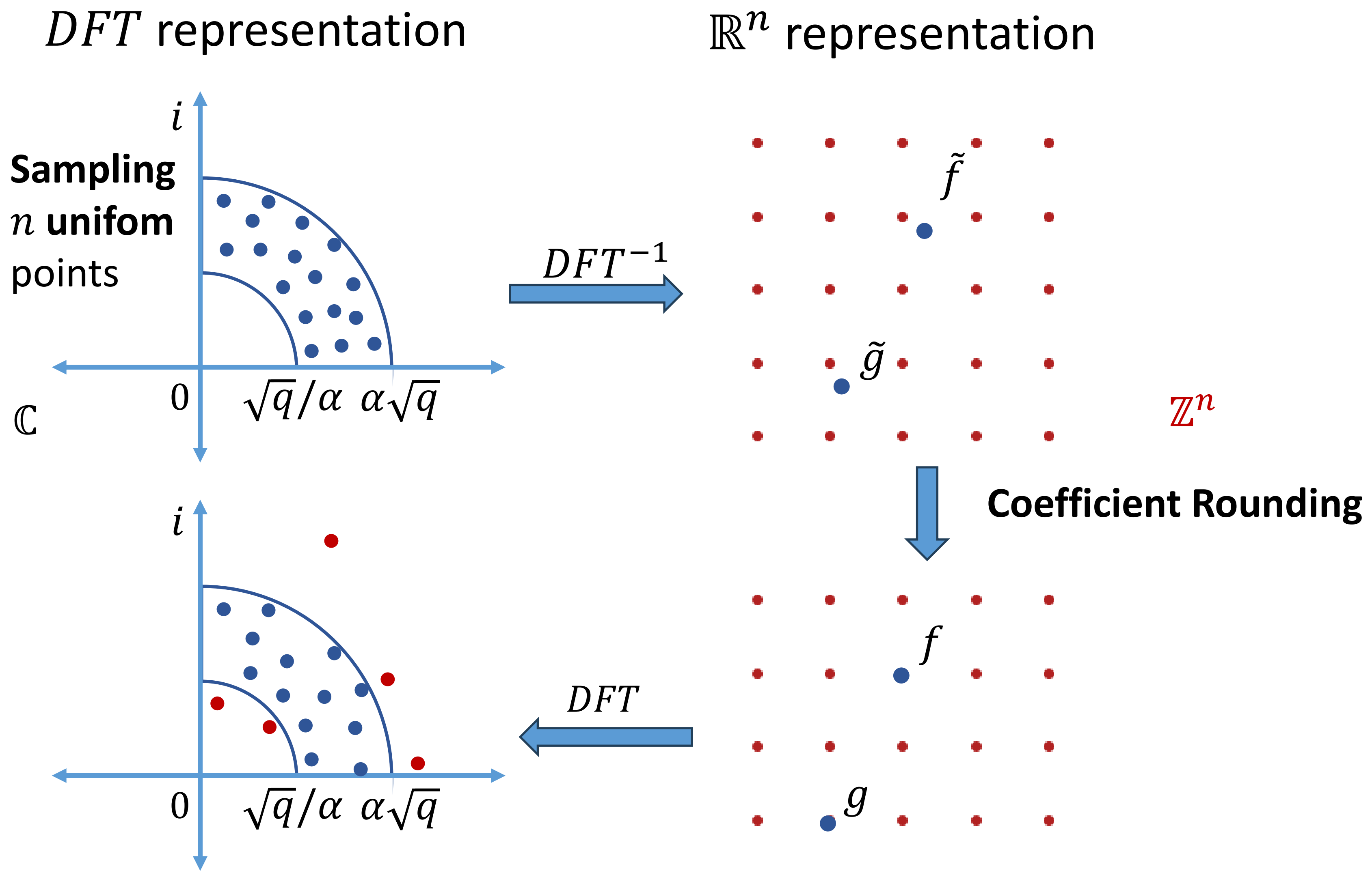

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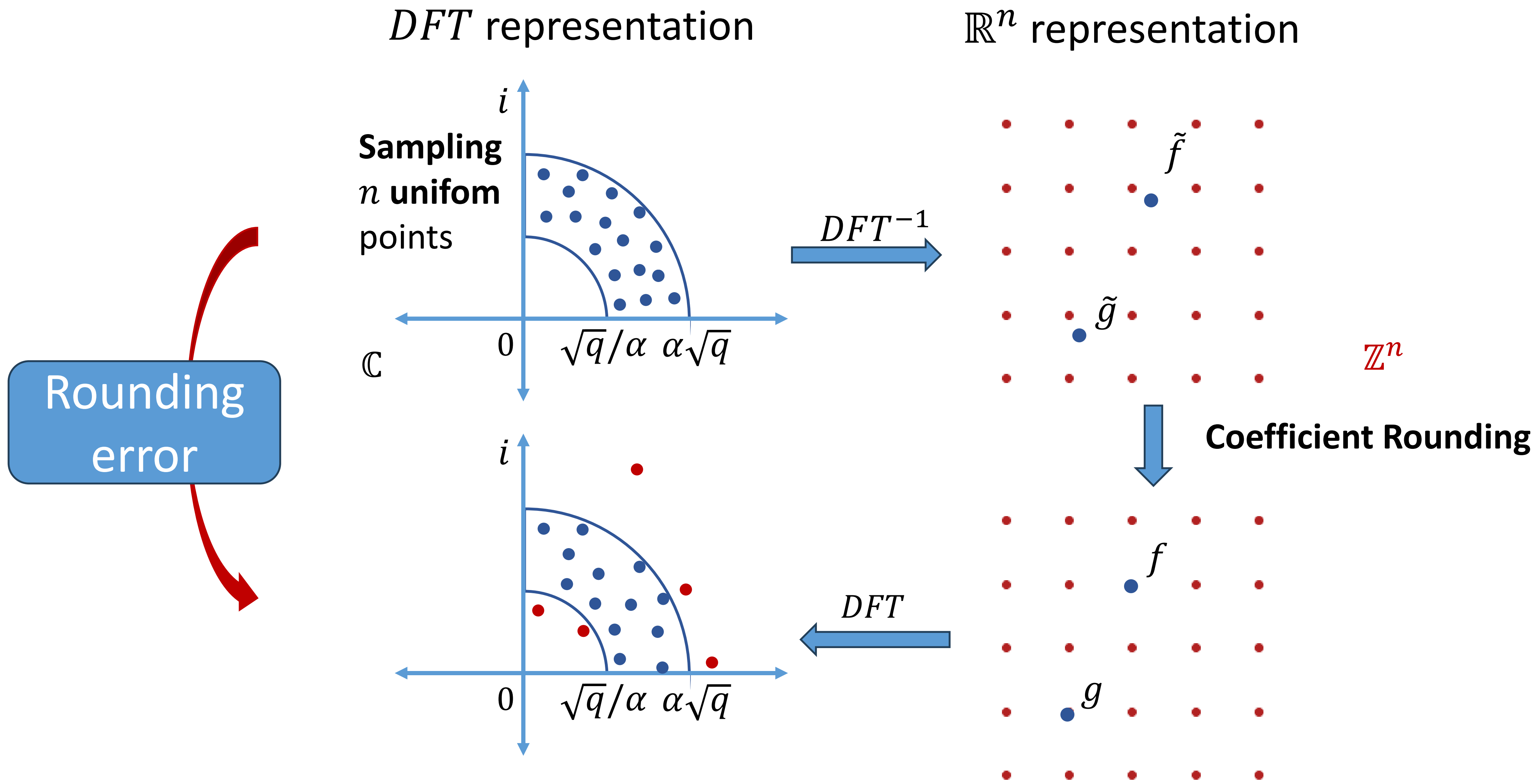
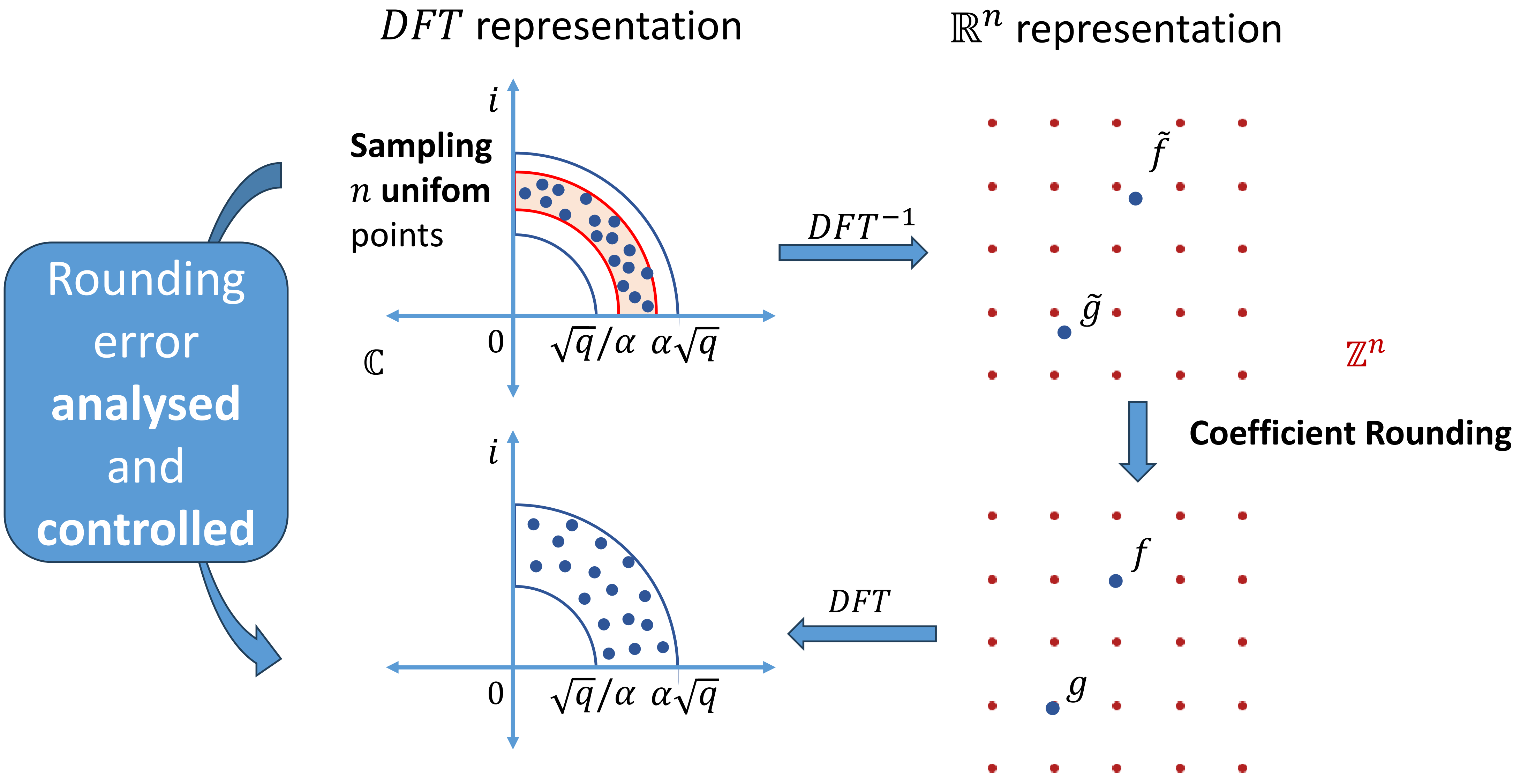
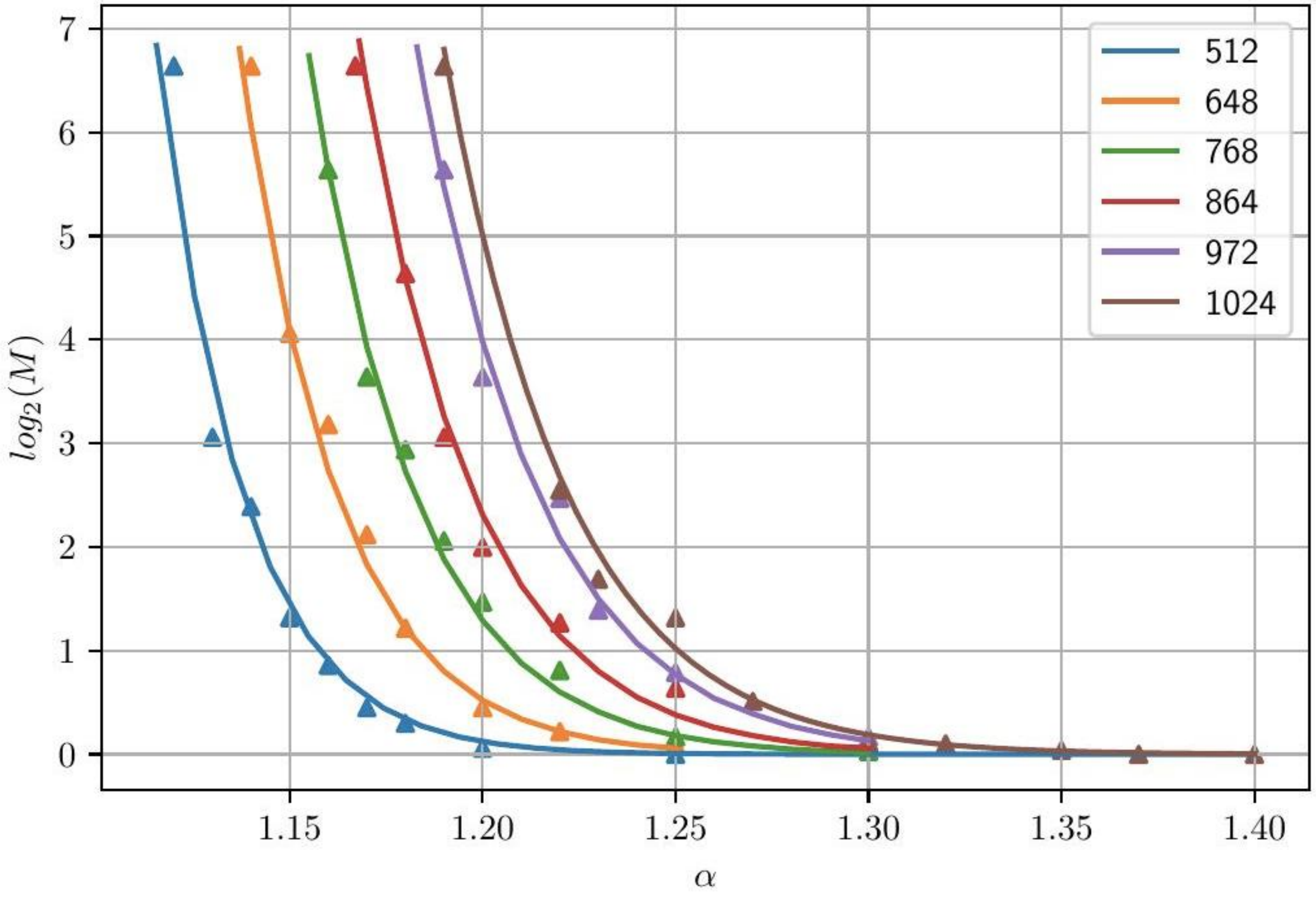


ANTRAG: Annular NTRU Trapdoor Generation (2)



Quality/repetition in ANTRAG



Performance comparison with Mitaka and Falcon

	Antrag+Hybrid	
n	**512**	**1024**
α	**1.15**	**1.23***
Keygen repetitions	**3**	**4**
Classical security (bits)	**124**	**264**
Sign speed (μs)	**8**	**15**
Signature size (bytes)	**646**	**1260**

Performance comparison with Mitaka and Falcon

	Antrag+Hybrid		Mitaka ($D_{\mathbb{Z}^n,0}$+Hybrid)	
n	**512**	**1024**	512	1024
α	**1.15**	**1.23***	2.04	2.33
Keygen repetitions	**3**	**4**	-	-
Classical security (bits)	**124**	**264**	102	233
Sign speed (μs)	**8**	**15**	8	16
Signature size (bytes)	**646**	**1260**	713	1405

- No precise number is given but Mitaka is estimated to have many repetitions.

Performance comparison with Mitaka and Falcon

	Antrag+Hybrid		Mitaka ($D_{\mathbb{Z}^n,0}$+Hybrid)		Falcon ($D_{\mathbb{Z}^n,0}$+FFO)	
n	**512**	**1024**	512	1024	512	1024
α	**1.15**	**1.23***	2.04	2.33	1.17	1.17
Keygen repetitions	**3**	**4**	-	-	8	8
Classical security (bits)	**124**	**264**	102	233	123	284
Sign speed (μs)	**8**	**15**	8	16	18	36
Signature size (bytes)	**646**	**1260**	713	1405	666	1280

*We do not need too small α to obtain the level NIST V of security.

- No precise number is given but Mitaka is estimated to have many repetitions.

3-smooth dimensions

n	648 $(2^3 \cdot 3^4)$			768 $(2^8 \cdot 3)$			864 $(2^5 \cdot 3^3)$			972 $(2^2 \cdot 3^5)$		
q	**12289**	**3889**	**9721**	**12289**	**3329**	**18433**	**12289**	**3727**	**10369**	**12289**	**4373**	**17497**
α	1.17	1.32	1.19	1.19	1.39	1.16	1.21	1.40	1.23	1.22	1.40	1.18
Repetitions	4	4	4	3	4	3	3	4	3	4	4	4
Classical/Quantum Security (bits)	166/ 151	159/ 144	164/ 149	196/ 178	192/ 174	195/ 177	222/ 201	220/ 200	222/ 201	251/ 227	254/ 230	250/ 227
Signature size (bytes)	808	747	796	952	883	977	1069	1000	1058	1701	1580	1225

Versatility with security!

Perspectives

- Antrag is integrated in the signature Solmae submitted at KPQC (Solmae = Antrag + Hybrid Sampler) (ongoing)
- More optimizations in Antrag's design (ongoing)
  - Annulus -> Circle sampling?
  - Integrating new rejection sampling technique
  - Full-fledged implementation?

Thank you!